

- 1 Prove that $\sin A \sin(60^\circ - A) \sin(60^\circ + A) = \frac{1}{4} \sin 3A$. [4m]
- 2 Given $\vec{AB} = (2k - 1)\mathbf{i} - \mathbf{j}$ and $\vec{AC} = -6\mathbf{i} + k\mathbf{j}$ are two vectors lie on a same plane. Find the value of k
- (a) if the points A, B and C are collinear, [4m]
- (b) if $\angle BAC = 90^\circ$.
- 3 The diagram shows a circle. PQ is a tangent to the circle at Q and the straight line PR cuts the circle at S. Show that
- (a) if $\angle RQS = \angle QPS$, then the length of chord QR is twice the radius of the circle, [3m]
- (b) if $\angle RQS = 30^\circ$, then the length of chord RS is equal to the radius of the circle. [3m]
- 4 Find the general solution of the differential equation $xy \frac{dy}{dx} + 5 - y^2 = 0$. [3m]
- Deduce that the particular solution for which $y = 1$ when $x = 2$ may be expressed in the form $x^2 + y^2 = 5$ and sketch the curve. [2m]
- Sketch, in separate diagram, the solution curve which passes through the point (1, 0). [3m]
- 5 In a parallelogram ABCD, X is the midpoint of AB and the line DX cuts the diagonal AC at P.
- Given that $\vec{AB} = \mathbf{a}$, $\vec{AD} = \mathbf{b}$, $\vec{AP} = \lambda \vec{AC}$ and $\vec{DP} = \mu \vec{DX}$, express \vec{AP} in terms of
- (a) λ , \mathbf{a} and \mathbf{b} . [2m]
- (b) μ , \mathbf{a} and \mathbf{b} . [3m]
- Hence, deduce that P is a point of trisection of both AC and DX. [5m]

- 6 Express $(\cos x + 1)^2 + (\sin x + \sqrt{3})^2$ in the form $a + b \cos(x - \alpha)$, where $a > 0$, $b > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Hence, [4m]
- (a) find the set of values of k such that the following equation has real roots $(\cos x + 1)^2 + (\sin x + \sqrt{3})^2 = k^2$, [3m]
- (b) solve, for $0 < x < 2\pi$, the equation $(\cos x + 1)^2 + (\sin x + \sqrt{3})^2 = 5 + 2\sqrt{2}$, [3m]
- (c) find the values of p and q such that $\frac{2}{(\cos x + 1)^2 + (\sin x + \sqrt{3})^2}$ lies in the interval (p, q) and the corresponding values of x. [5m]
- 7 The result of a veterinary survey on 125 rabbits, each suspected of having a particular disease is summarize in the following table.
- | | Diseased | Not Diseased | Total |
|--------|----------|--------------|-------|
| Female | 25 | 35 | 60 |
| Male | 20 | 45 | 65 |
| Total | 45 | 80 | 125 |
- The case history of each rabbit was kept on a separate record card. If three different record cards are selected at random, without replacement, calculate the probability that
- (a) all three record cards relate to rabbits with the disease, [3m]
- (b) one record card relates to a female rabbit with the disease, one to a male rabbit with the disease and one to a female rabbit not suffering from the disease. [3m]
- 8 The probability that a shooter strikes a target in one shot is 0.7. Find the probability that not less than three of five shots fired, strike the target. [3m]
- After the shooter has completed a one month shooting course, the probability that he strikes the target in one shot is 0.95. By using a suitable approximation, find the probability that in a sample of 50 shots, more than 45 shots strike the target. [4m]

9 Three red balls and three white balls are placed in a box. Balls are drawn one by one, at random and without replacement. The random variable X is the number of white balls drawn before the first red ball is drawn.

(a) Show that $P(X = 1) = \frac{3}{10}$, and find the rest of the probability distribution function of X . [3m]

(b) Find $E(X)$ and show that $\text{Var}(X) = \frac{63}{80}$. [4m]

10 The table shows the durations of 60 journeys on the same route by a lorry, the variations in journey times being caused by varying traffic conditions.

Time of journey in hours	Number of journeys
4.6–4.8	2
4.8–5.0	7
5.0–5.2	16
5.2–5.4	21
5.4–5.6	12
5.6–5.8	2

Calculate, to the nearest minute, estimates of the mean and standard deviation for the duration of the journeys. [4m]

When the times for 40 other journeys were taken, it was found that the mean and standard deviation for the times of these 40 journeys were 5 hr 24 min and 18 min, respectively. Find also, to the nearest minutes, the estimated mean and standard deviation for the duration of all 100 journeys. [5m]

11 The continuous random variable X is the distance, measured in hundreds of kilometers, that a particular car will travel on a full tank of petrol. It is given that

$$P(X \leq x) = \begin{cases} 0, & x \leq 4, \\ ax^2 - 10ax - 24, & 4 \leq x \leq 5, \\ 1, & 5 \leq x \end{cases}$$

where a is a constant. Show that $a = -1$. [2m]

(a) Find the probability density function of X . [3m]

(b) Show that $E(X) = \frac{13}{3}$. [3m]

(c) Three independent observations of X are taken. Find the probability that two of the observations are less than 4.5 and one is greater than 4.5. [3m]

12 A factory produces both metal rods and tubes. The internal diameter, in cm, of a metal tube is distributed $N(2.2, 0.0009)$. The diameter, in cm, of a metal rod is distributed $N(2.15, 0.0004)$.

(a) Find the percentage of tubes with internal diameter less than 2.14 cm. [3m]

(b) Find the percentage of rods with diameter greater than 2.1 cm and less than 2.2cm. [3m]

(c) A rod and a tube are chosen at random. Find the probability that the rod will not pass through the tube. [5m]

(d) Three packets, each containing a rod and a tube, are chosen at random.

Find the probability that two packets contain rods that will pass through their respective tubes and the other will not. [3m]