1. LHS $= \sin A \sin(60° - A)\sin(60° + A)$

$= \sin A\left[-\frac{1}{2}(\cos 120° - \cos 2A)\right]$ M1

$= \sin A\left(-\frac{1}{2}\right)\left[-\frac{1}{2} - \cos 2A\right]$ A1

$= \frac{1}{2}\sin A\left[\frac{1}{2} + 1 - 2\sin^2 A\right]$ M1

$= \frac{1}{4}\sin A\left[3 - 4\sin^2 A\right]$

$= \frac{1}{4}\left[3\sin A - 4\sin^3 A\right]$ A1

$= \frac{1}{4}\sin 3A$

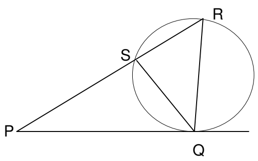
2. (a) $\frac{-1}{2k-1} = \frac{k}{-6}$ or vector method M1

$k = -\frac{3}{2}$ or $k = 2$ A1

(b) $\left(\frac{-1}{2k-1}\right) \times \left(\frac{k}{-6}\right) = -1$ or vector method M1

$k = \frac{6}{13}$ A1

3.



(a) Given $\angle RQS = \angle QPS = \alpha$

$\angle QRS = \angle PQS = \beta$ (Angle in alternate segments) B1

$2\alpha + 2\beta = 180°$ (Sum angles of ΔPQR)

$\alpha + \beta = 90°$.

Therefore $\angle QSR = 180° - \alpha - \beta = 90°$ (Sum angles of ΔQRS)

$\Rightarrow$ Chord QR is the diameter of the circle (Angle in semi-circle) B1

The length of chord QR is twice the radius of the circle. B1

(b) Let O be the centre of circle

$\angle ROS = 60°$ (Angle at the centre is twice angle at the circumference) B1

OR = OS (radius of circle)

ΔROS is isosceles

$\angle OSR = \angle ORS = 60° = \angle ROS$

ΔORS is an equilateral triangle B1

Therefore OR = OS = RS

Length of RS equal to the radius B1

4. $xy\frac{dy}{dx} + 5 - y^2 = 0.$

$\int \frac{y}{y^2-5}\,dy = \int \frac{1}{x}\,dx$ B1

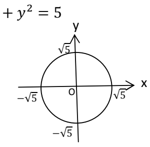
$\frac{1}{2}\ln(y^2 - 5) = \ln Ax$ M1

$y^2 - 5 = Cx^2$ A1

$x = 2, y = 1 \Rightarrow C = -1$ M1

$y^2 - 5 = -x^2$

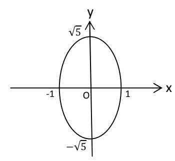
$x^2 + y^2 = 5$ A1



D1

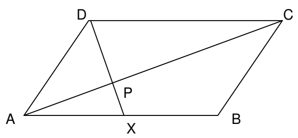
$x = 1, y = 0 \Rightarrow C = -5$

$5x^2 + y^2 = 5$ M1



A1

5.



(a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$ M1

$\overrightarrow{AP} = \lambda\overrightarrow{AC} = \lambda(\mathbf{a} + \mathbf{b})$ A1

(b) $\overrightarrow{AP} = \overrightarrow{AD} + \overrightarrow{DP}$

$= \mathbf{b} + \mu\overrightarrow{DX}$ M1

$= \mathbf{b} + \mu\left(\frac{1}{2}\mathbf{a} - \mathbf{b}\right)$ M1

$= \frac{1}{2}\mu\mathbf{a} + (1-\mu)\mathbf{b}$ A1

Comparing; $\lambda(\mathbf{a} + \mathbf{b}) = \frac{1}{2}\mu\mathbf{a} + (1-\mu)\mathbf{b}$, we get M1

$\lambda = \frac{1}{2}\mu$ and $\lambda = 1 - \mu$ M1

Solving: $\lambda = \frac{1}{3}$ and $\mu = \frac{2}{3}$ A1 A1

$\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AC}$ and $\overrightarrow{DP} = \frac{2}{3}\overrightarrow{DX}$ or equivalent ly $\overrightarrow{XP} = \frac{1}{3}\overrightarrow{XD}$

Thus, the point P trisects AC and XD. A1

6. $(\cos x + 1)^2 + (\sin x + \sqrt{3})^2 = 5 + 2[\cos x + \sqrt{3}\sin x]$

$= 5 + 2[b\cos(x - \alpha)]$ M1

$b = 2, \quad \alpha = \frac{\pi}{3}$ M1M1

$= 5 + 4\cos\left(x - \frac{\pi}{3}\right)$ A1

(a) $(\cos x + 1)^2 + (\sin x + \sqrt{3})^2 = k^2$

$5 + 4\cos\left(x - \frac{\pi}{3}\right) = k^2$

$\cos\left(x - \frac{\pi}{3}\right) = \frac{k^2 - 5}{4}$

$-1 \le \frac{k^2 - 5}{4} \le 1$ M1

$1 \le k^2 \le 9$

$\{k : k \in R : -3 \le k \le -1 \text{ or } 1 \le k \le 3\}$ A1A1

(b) $(\cos x + 1)^2 + (\sin x + \sqrt{3})^2 = 5 + 2\sqrt{2}$

$\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$ M1

$x - \frac{\pi}{3} = -\frac{\pi}{4}, \frac{\pi}{4}$ M1

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ A1

(c) $1 \le 5 + 4\cos\left(x - \frac{\pi}{3}\right) \le 9$ M1

$\frac{2}{9} \le \frac{2}{5 + 4\cos\left(x - \frac{\pi}{3}\right)} \le 2$ M1

$p = \frac{2}{9}$ and $q = 2$ A1

$p = \frac{2}{9}$, corresponding value of $x = \frac{\pi}{3}$ B1

$q = 2$, corresponding value of $x = \frac{4\pi}{3}$ B1

7. $n(S) = {}^{125}C_3$ B1

(a) Required probability $= \frac{{}^{45}C_3}{{}^{125}C_3}$ M1

$= 0.0447$ A1

(b) Required probability $= \frac{{}^{25}C_1\,{}^{20}C_1\,{}^{35}C_1}{{}^{125}C_3}$ B1M1

= 0.0551 A1

8. (a) $P(X = 3) + P(X = 4) + P(X = 5)$ Three correct terms M1

$= 10(0.7)^3(0.3)^2 + 5(0.7)^4(0.3) + (0.7)^5$. Binomial distribution M1

= 0.8369 A1

(b) P(he miss the target) = P(M) = 0.05

np = 2.5 M1

$P(M \le 4) = P(M = 0) + P(M = 1) + P(M = 2) + P(M = 3) + P(M = 4)$

sum of 5 terms M1

$= e^{-2.5}\{1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{6} + \frac{2.5^4}{24}\}$

Use Poisson distribution M1

= 0.8912 A1

9. (a) P(X = 1) = P(WR)

$= \frac{3}{6} \times \frac{3}{5}$

$= \frac{3}{10}$ B1

$P(X = 0) = P(R) = \frac{3}{6} = \frac{1}{2}$

$P(X = 2) = P(WWR) = \frac{3}{6} \times \frac{2}{5} \times \frac{3}{4} = \frac{3}{20}$

$P(X = 3) = P(WWWR) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3} = \frac{1}{20}$ B1B1

(b) $E(X) = 0 + \frac{3}{10} + 2\left(\frac{3}{20}\right) + 3\left(\frac{1}{20}\right)$ M1

$= \frac{3}{4}$ A1

$E(X^2) = 0 + \frac{3}{10} + 4\left(\frac{3}{20}\right) + 9\left(\frac{1}{20}\right) = \frac{27}{20}$

$\therefore Var(X) = \left(\frac{27}{20}\right) - \left(\frac{3}{4}\right)^2$ M1

$= \frac{63}{80}$ A1

10.

x	*f*
4.7	2
4.9	7
5.1	16
5.3	21
5.5	12
5.7	2

Mean $= \frac{\sum fx}{\sum f}$

$= \frac{311.6}{60}$ M1

= 5 hr 12 minutes A1

S. deviaton $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

$= \sqrt{\frac{1620.36}{60} - \left(\frac{311.6}{60}\right)^2}$ M1

= 11 minutes A1

New mean $= \frac{311.6 + 5.6 \times 40}{100}$ M1

= 5 hr 21 minutes A1

$\sqrt{\frac{\sum fx^2}{40} - (5.6)^2} = 0.3$

$\therefore \sum fx^2 = 1258$ M1

New s. deviation $= \sqrt{\frac{1620.36 + 1258}{100} - \left(\frac{311.6 + 5.6 \times 40}{100}\right)^2}$ M1

= 19 minutes A1

11. $P(X \le 4) = 0$

$\therefore a(16) - 40a - 24 = 0$ M1

$\therefore a = -1$ A1

(a) $f(x) = \begin{cases} -2x + 10, & 4 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$ M1A1

(b) $E(X) = \int_4^5 x(-2x + 10)\, dx$ B1

$= \int_4^5 -2x^2 + 10x \, dx$

$= \left[-\frac{2x^3}{3} + 5x^2\right]_4^5$ M1

$= \frac{13}{3}$ A1

(c) $P(X<4.5) = -(4.5)^2 + 10(4.5) - 24 = \frac{3}{4}$ B1

Prob $= \frac{3!}{2!}[P(X<4.5)]^2[P(X>4.5)]$

$= \frac{3!}{2!} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$ M1

$= \frac{27}{64}$ A1

12.

(a) Probability $= P(X < 2.14)$

$= P\left(Z < \frac{2.14 - 2.2}{0.03}\right)$ M1

$= P(Z < -2)$ A1

= 0.0228

Percentage = 2.28 % A1

(b) Probability $= P(2.1 < Y < 2.2)$

$= P\left(\frac{2.1 - 2.15}{0.02} < Z < \frac{2.2 - 2.15}{0.02}\right)$ M1

$= P(-2.5 < Z < 2.5)$ A1

= 0.9876

Percentage = 98.8% A1

(c) $X - Y \sim N(2.2 - 2.15,\ 0.03^2 + 0.02^2)$

$X - Y \sim N(0.05,\ 0.0013)$ B1

P (rod will not pass through tube)

$= P(X < Y)$ B1

$= P(X - Y < 0)$

$= P\left(Z < \frac{0 - 0.05}{\sqrt{0.0013}}\right)$ M1

$= P(Z < -1.387)$ A1

= 0.08272 ≈ 0.0827 A1

(d) P (rod will pass through tube) = 1 – 0.0827 = 0.9173 B1

P (two packets out of three selected packets of rod –tube where rod will pass through tube)

$= 3(0..9173)^2(0.0827)$ M1

= 0.2088 ≈ 0.209 A1