1. Given that $\log_2 P = x$, $\log_3 P = y$ and $x + y = 1$, show that $x = \dfrac{\ln 3}{\ln 6}$. [4 marks]

2. Prove that $\displaystyle\sum_{r=0}^{n} \frac{1}{4r^2-1} = -\frac{n+1}{2n+1}$. [4 marks]

3. Show, by means of the substitution $x = \tan\theta$, that

$$\int_0^1 \frac{1}{\left(1+x^2\right)^2}\,dx = \int_0^{\frac{\pi}{4}} \cos^2\theta\,d\theta .$$

Hence, find the exact value of $\displaystyle\int_0^1 \frac{1}{\left(1+x^2\right)^2}\,dx$. [6 marks]

4. In the same diagram, sketch the graph $y = |x-2|$, $x \in \Re$ and the graph $y = \sqrt{x}, x \geq 0$.

Hence or otherwise solve the inequality $|x-2| < \sqrt{x}$. [6 marks]

5. Given that $y = \dfrac{\cos(ax)}{x^3}$, where a is a constant and $x \neq 0$, show that

$$x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + \left(6 + a^2x^2\right)y = 0 .$$ [6 marks]

6. A straight line l_1 with gradient m, passes through the point (4, 0) is parallel to the line l_2, which passes through the point (-4, 0). l_1 and l_2 meet the line $4x + 5y = 25$ at point F and at point G respectively.

(a) Find the coordinates of point F and point G, in terms of m. [5 marks]

(b) If the distance of FG is 8 units, find the possible values of m. [3 marks]

7. The complex numbers z and w are given by z = 3 + 2i and w = –5 + 4i.

(a) Find |w| in surd form and arg w in radians correct to three significant figures. [3 marks]

(b) Express $\dfrac{z}{w}$ in the form $a + ib$, where a and b are exact fractions. [2 marks]

(c) In an Argand diagram, the points Z and W represent the complex numbers z and w respectively, whereas the point Z* represents z^*, the conjugate of z. The point P is such that ZWZ*P (in that order) is a parallelogram. Find the complex number p represented by point P. [3 marks]

8. If $y = 5^{2x}$, by taking logarithm to the base e for both sides of the equation, show that $\dfrac{dy}{dx} = 5^{2x}\ln 25$. [2 marks]

Hence, determine the integers m and n for which $\displaystyle\int_0^1 x5^{2x}(\ln 25)^2\,dx = m\ln 5 - n$. [7 marks]

9. Function f and g are defined by

$f : x \to a(x+3)^2 - b$ for $x \leq -3$, where a and b are constants.

$g : x \to \sqrt{\dfrac{x+5}{2}}$

(a) State the domain and range of g. [2 marks]

(b) Find the values of a and b if $g \circ f(x) = -(x+3)$, $x \leq -3$. [3 marks]

(c) Sketch the graph of f and explain why the inverse function of f exists, hence find $f^{-1}(x)$. [5 marks]

10. Matrix P is given by $P = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

If $P^2 + aP + bI = \begin{pmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix}$, where I is the 3 × 3 identity matrix, find the values of a and b. [4 marks]

Find $P(P^2 + aP + bI)$. [1 mark]

Hence, solve the simultaneous equations

$$x - z = 3$$
$$x + 2y + z = 2$$
$$2x + 2y + 3z = 5$$ [5 marks]

11. Find the value of a and of b if $f(x) = x^4 + ax^3 + 13x^2 - 12x + b$ is exactly divisible by $g(x) = x^2 - 3x + 2$. [2 marks]

Hence,

(a) find the solution of $f(x) = 0$. [3 marks]

(b) find the set of values of x which satisfy $f(x) \leq -3g(x)$. [5 marks]

(c) using the substitution $y = \frac{1}{x}$, solve the equation

$$4y^4 - 12y^3 + 13y^2 - 6y + 1 = 0$$ [3 marks]

12. Given that $f(x) = 2x^2 + \ln(4x + 5)$, has domain $\{x : x \in R, -\frac{5}{4} < x \leq 1\}$.

(a) State the asymptote of f. [1 mark]

(b) Find all (local) maximum and minimum points of f. [7 marks]

(c) Find the coordinate of the point of inflexion. [3 marks]

(d) Sketch the graph of f. [4 marks]

(e) State the maximum value of f for the given domain. [1 mark]