STPM KEDAH 2012-MT PAPER 1-MARKING SCHEME

1.

$2^x = 3^y$ B1

$2^x = 3^{1-x}$

$x \ln 2 = (1 - x)\ln 3$ M1A1

$x(\ln 2 + \ln 3) = \ln 3$

$x = \dfrac{\ln 3}{\ln 6}$. A1

2.

$\sum_{r=0}^{n} \frac{1}{4r^2 - 1}$

$= \frac{1}{2}\sum_{r=0}^{n}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$ B1

$= \frac{1}{2}[(-1-1) + (1 - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{5}) + \cdots + \left(\frac{1}{2n-5} - \frac{1}{2n-3}\right) + \left(\frac{1}{2n-3} - \frac{1}{2n-1}\right)$ M1

$+ \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$

$= \frac{1}{2}\left[-1 - \frac{1}{2n+1}\right] = -\frac{n+1}{2n+1}$. (proved) M1A1

3.

$\frac{dx}{d\theta} = \sec^2\theta$, B1

$\tan 0 = 0$, $\tan\frac{\pi}{4} = 1$

$(1 + x^2)^2 = (1 + \tan^2\theta)^2 = \sec^4\theta$ M1

$\int_0^1 \frac{1}{(1+x^2)^2}\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{\sec^4\theta}\cdot\sec^2\theta\, d\theta$ A1

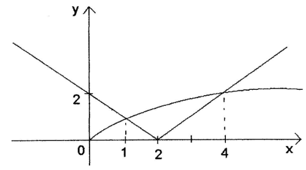
$= \int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$

$\int_0^1 \frac{1}{(1+x^2)^2}\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{2}(1+\cos 2\theta)\,d\theta$ M1

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{4}}$ A1

$= \frac{1}{8}[\pi + 2]$. A1

4.



D1: $y = \sqrt{x}$

D1 : shape of $y = |x - 2|$

D1 : points (2, 0) , (0, 2)

B1 : x = 1 , 4

B1 : 1 < x < 4

The solution set is $\{x : 1 < x < 4, x \in \Re\}$ B1

5.

$y = x^{-3}\cos ax$

$yx^3 = \cos ax$ ——— (1)

$y(3)(x^2) + x^3\frac{dy}{dx} = -a\sin ax$ M1A1

$3x^2 y + x^3\frac{dy}{dx} = -a\sin ax$

$3\left[x^2\frac{dy}{dx} + y(2)(x)\right] + x^3\frac{d^2y}{dx^2} + \frac{dy}{dx}(3)(x^2) = -a^2\cos ax$ M1A1

$3x^2\frac{dy}{dx} + 6xy + x^3\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} = -a^2\cos ax$

$6x^2\frac{dy}{dx} + 6xy + x^3\frac{d^2y}{dx^2} = -a^2\cos ax$ ——— (2)

Sub (1) into (2)

$6x^2\frac{dy}{dx} + 6xy + x^3\frac{d^2y}{dx^2} = -a^2 yx^3$ M1

$6x^2\frac{dy}{dx} + 6xy + x^3\frac{d^2y}{dx^2} + a^2yx^3 = 0$

$(x)\left(x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + 6y + a^2x^2y\right) = 0$

$x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + (6 + a^2x^2)y = 0$ A1

6.

Let $\left.\begin{matrix} l_1 : y = m(x-4) \\ l_2 : y = m(x+4) \end{matrix}\right\}$ B1 (l_1 or l_2)

intercept $4x + 5y = 25$ at point $F(x_1, y_1)$ and $G(x_2, y_2)$.

(a)

$4x_1 + 5[m(x_1 - 4)] = 25$ M1

$4x_1 + 5mx_1 - 20m = 25$

$x_1(4 + 5m) = 25 + 20m$

$x_1 = \frac{25 + 20m}{4 + 5m}$ A1

$y_1 = m\left(\frac{25 + 20m}{4 + 5m} - 4\right)$

$y_1 = \frac{9m}{4 + 5m}$

$\therefore\ F\left(\frac{25+20m}{4+5m}, \frac{9m}{4+5m}\right)$ A1

$4x_2 + 5[m(x_2 + 4)] = 25$

$4x_2 + 5mx_2 - 20m = 25$

$x_2(4 + 5m) = 25 + 20m$

$x_2 = \frac{25 - 20m}{4 + 5m}$

$y_2 = m\left(\frac{25 - 20m}{4 + 5m} + 4\right)$

$y_2 = \frac{41m}{4 + 5m}$

$\therefore\ G\left(\frac{25-20m}{4+5m}, \frac{41m}{4+5m}\right)$ A1

(b)

$\sqrt{\left[\frac{25+20m}{4+5m} - \left(\frac{25-20m}{4+5m}\right)\right]^2 + \left[\frac{9m}{4+5m} - \left(\frac{41m}{4+5m}\right)\right]^2} = 8$ M1 ✓

$\left(\frac{40m}{4+5m}\right)^2 + \left(\frac{-32m}{4+5m}\right)^2 = 8^2$

$\frac{2624m^2}{16 + 40m + 25m^2} = 64$

$16m^2 + 40m + 16 = 0$

$2m^2 + 5m + 2 = 0$

$(2m + 1)(m + 2) = 0$ M1

$m = -\frac{1}{2}, -2$ A1

7.

(a) $|w| = \sqrt{41}$, B1

$\arg w = \tan^{-1}\left(\frac{4}{-5}\right) = 2.47$ rad. (3 s.f.) M1A1

(b) $\frac{z}{w} = \frac{3 + 2i}{-5 + 4i} \times \frac{-5 - 4i}{-5 - 4i} = -\frac{7}{41} - \frac{22}{41}i$ M1A1

(c) Z(3, 2), W(–5, 4), Z*(3, –2), P(x, y)

mid-pt of ZZ* = mid-pt of WP

$(3, 0) = \left(\frac{x-5}{2}, \frac{y+4}{2}\right)$ M1

$x = 11$, $y = -4$ A1

$p = 11 - 4i$. A1

8.

$\ln y = 2x\ln 5$

$\frac{1}{y}\frac{dy}{dx} = 2\ln 5$ M1

$\frac{dy}{dx} = 2y\ln 5$ A1

$\frac{dy}{dx} = 5^{2x}\ln 25$

$\int_0^1 x5^{2x}(\ln 25)^2\,dx$

$= \int_0^1 (x\ln 25)5^{2x}\ln 25\,dx$

$= \left[x5^{2x}\ln 25 - \int 5^{2x}\ln 25\,dx\right]_0^1$ B1B1

$= \left[x5^{2x}\ln 25 - 5^{2x}\right]_0^1$ M1

$= 50\ln 5 - 24$ M1A1

Hence, $m = 50$, $n = 24$ A1A1

9.

(a) Domain : $g = \{x : x \in \Re, x \geq -5\}$ B1

Range : $g = \{y : y \in \Re, y \geq 0\}$ B1

(b) $g \circ f(x) = -(x+3)$

$$\sqrt{\frac{a(x+3)^2 - b + 5}{2}} = -(x+3)$$

$$\frac{a(x+3)^2 - b + 5}{2} = [-(x+3)]^2$$ M1

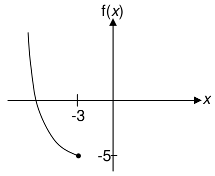
$$\frac{a}{2} = 1$$

$a = 2$ A1

$$\frac{-b+5}{2} = 0$$

$b = 5$ A1

(c)



Shape D1
All correct D1

Since the line parallel to the x-axis intersects the curve y = f(x) once, therefore the function *f* is one-to-one function. B1

$f(x) = 2(x+3)^2 - 5$

$2(y+3)^2 - 5 = x$ M1

$2(y+3)^2 = x + 5$

$$(y+3)^2 = \frac{x+5}{2}$$

$$y + 3 = \pm\sqrt{\frac{x+5}{2}}$$

$$y + 3 = -\sqrt{\frac{x+5}{2}} \quad \text{since} \quad y \le -3$$

$$y = -3 - \sqrt{\frac{x+5}{2}}$$

$$\therefore f^{-1}(x) = -3 - \sqrt{\frac{x+5}{2}}, x \ge -5$$ A1

10. $$\begin{pmatrix}1&0&-1\\1&2&1\\2&2&3\end{pmatrix}\begin{pmatrix}1&0&-1\\1&2&1\\2&2&3\end{pmatrix} + a\begin{pmatrix}1&0&-1\\1&2&1\\2&2&3\end{pmatrix} + b\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = \begin{pmatrix}4&-2&2\\-1&5&-2\\-2&-2&2\end{pmatrix}$$

$$\begin{pmatrix}-1&-2&-4\\5&6&4\\10&10&9\end{pmatrix} + \begin{pmatrix}a&0&-a\\a&2a&a\\2a&2a&3a\end{pmatrix} + \begin{pmatrix}b&0&0\\0&b&0\\0&0&b\end{pmatrix} = \begin{pmatrix}4&-2&2\\-1&5&-2\\-2&-2&2\end{pmatrix}$$ M1

$$\begin{pmatrix}-1+a+b&-2&-4-a\\5+a&6+2a+b&4+a\\10+2a&10+2a&9+3a+b\end{pmatrix} = \begin{pmatrix}4&-2&2\\-1&5&-2\\-2&-2&2\end{pmatrix}$$ A1

$5 + a = -1$
$a = -6$ A1

$-1 + a + b = 4$
$b = 11$ A1

$$P(P^2 + aP + bI) = \begin{pmatrix}1&0&-1\\1&2&1\\2&2&3\end{pmatrix}\begin{pmatrix}4&-2&2\\-1&5&-2\\-2&-2&2\end{pmatrix}$$

$$= \begin{pmatrix}6&0&0\\0&6&0\\0&0&6\end{pmatrix}$$ B1

$P(P^2 + aP + bI) = 6I$

$$P^{-1} = \frac{1}{6}\begin{pmatrix}4&-2&2\\-1&5&-2\\-2&-2&2\end{pmatrix}$$ B1

$$\begin{pmatrix}1&0&-1\\1&2&1\\2&2&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\2\\5\end{pmatrix}$$ B1

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{1}{6}\begin{pmatrix}4&-2&2\\-1&5&-2\\-2&-2&2\end{pmatrix}\begin{pmatrix}3\\2\\5\end{pmatrix}$$ M1

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\-\frac{1}{2}\\0\end{pmatrix}$$ A1

$x = 3, y = -\frac{1}{2}, z = 0$ A1

11. $f(1) = a + b + 2 = 0$

$f(2) = 8a + b + 44 = 0$ M1

$\therefore a = -6, b = 4$ A1

(a) $x^4 - 6x^3 + 13x^2 - 12x + 4 = 0$

$(x^2 - 3x + 2)(x^2 - 3x + 2) = 0$ M1A1

$(x-1)^2(x-2)^2 = 0$

$x = 1, x = 2$ A1

(b) $f(x) \le -3g(x)$

$(x-1)^2(x-2)^2 + 3(x-1)(x-2) \le 0$ M1A1

$(x-1)(x-2)(x^2 - 3x + 5) \le 0$

Consider $x^2 - 3x + 5$,

Since $a = 1, D = -11 < 0$ M1

Hence $x^2 - 3x + 5 \ge 0, x \in R$ A1

Hence, $(x-1)(x-2) \le 0$ A1

$\therefore 1 \le x \le 2$

The solution set is $\{x: 1 \le x \le 2, x \in R\}$

c) $4y^4 - 12y^3 + 13y^2 - 6y + 1 = 0$

$$4\left(\frac{1}{x}\right)^4 - 12\left(\frac{1}{x}\right)^3 + 13\left(\frac{1}{x}\right)^2 - 6\left(\frac{1}{x}\right) + 1 = 0$$

$x^4 - 6x^3 + 13x^2 - 12x + 4 = 0$ B1

$(x-1)^2(x-2)^2 = 0$ M1 refer to (a)

$x = 1, x = 2$ A1

$y = 1, y = \frac{1}{2}$

12. (a) the asymptote is $x = -\frac{5}{4}$. B1

(b) $f'(x) = 4x + \frac{4}{4x+5}$. M1 for f' and f''

$$= \frac{4(4x+1)(x+1)}{4x+5}$$

$f''(x) = 4 + 4(-1)(4x+5)^{-2}(4)$

$$= \frac{4(4x+7)(4x+3)}{(4x+5)^2}$$ A1 for f' or f''

When $f'(x) = 0$, $(x, y) = (-1, 2)$ or $(-\frac{1}{4}, \frac{1}{8} + \ln 4)$ M1 A1 for stationary points

x	$-\frac{5}{4}$ to -1	-1		$-\frac{3}{4}$		$-\frac{1}{4}$	
sign of $f'(x)$:	+	o	−		−	o	+
sign of $f''(x)$:	−		−	o	+		+

M1 determine nature of stationary points

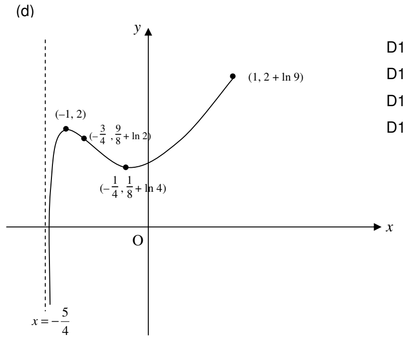
local maximum at $(-1, 2)$ A1

local minimum at $(-\frac{1}{4}, \frac{1}{8} + \ln 4)$ A1

(c) When $f''(x) = 0$, $x = -\frac{3}{4}$ $\{-\frac{7}{4}$ is not in the given domain$\}$ M1 A1

inflexion at $(-\frac{3}{4}, \frac{9}{8} + \ln 2)$ A1

(d)



D1 for shape
D1 for asymptote
D1 for points in (b) & (c)
D1 for end point (1, 2 + ln 9)

(e) maximum *f* occurs at $x = 1$
max. *f* is $2 + \ln 9$ B1